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High-Speed Rotor Dynamics—An Assessment of Current Technology for Small Turboshaft Engines

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An extensive study was made to determine current needs for research in rotor dynamics to solve problems encountered in small high-speed turboshaft engines for helicopter and aircraft propulsion. The purpose of this paper is to report the state-of-the-art for this area as completely and concisely as possible. The present and past philosophy of rotor-bearing system design including the impact of the demand for front drives, is discussed. Methods for critical speed prediction and high-speed balancing are reviewed. The trend to higher speeds is seen to require consideration of new approaches to balancing through flexural modes. The major parameters available for control by the designer are shown to be the bearing support properties, and recommendations are made for improving the accuracy of prediction of these properties. Nonsynchronous excitation is categorized according to the mechanisms producing the forces, and a need is shown for better methods to identify the resulting whirling and vibration, since several of these motions are potentially unstable. Finally, reasons are given for the predominant use of rolling-element bearings in these engines, and the potential for special applications of oil-film and gas bearings is discussed.

I. Introduction

MODERN design of small turboshaft engines is characterized by ever increasing power weight and power/size ratios. The increase in performance is being obtained in part from higher shaft speeds, which has heightened the importance of rotor dynamics considerations in the design and development process.

In particular, there are a number of problem areas connected with rotor dynamics which are peculiar to the special requirements of rotor-bearing systems of small turboshaft engines for helicopter or aircraft propulsion. These special requirements are: 1) Increasingly higher shaft speeds. Increased airflow and power output can thus be obtained without an increase in physical size requirements. 2) Small frontal area and light weight. 3) Front drive. It is usually desired to locate the power takeoff shaft out through the front of the compressor section. 4) Maintainability. Individual components making up the rotor-bearing assembly should be easily replaceable. 5) Long life. It is desired to increase the engine operating life significantly. Frequency of overhauls should be reduced.

Even if only taken one at a time on an individual basis, these requirements would generate significant problems for the rotor-bearing design engineer. Taken together simultaneously, these requirements are a severe challenge to meet. For example, the combination of requirements 1 and 2 has resulted in a blade tip clearance problem in small engines. The very short length blades require extremely small tip to housing clearances to maintain high aerodynamic efficiency and high power output. This in turn requires very small rotor shaft excursions to avoid blade-housing interferences, a design condition which is incompatible with low dynamic bearing loads at high speeds.

An extensive study was made by the authors for the U.S. Army (USAAMRDL, Eustis Directorate, Propulsion Technical Area) to determine the current needs for research in rotor dynamics. From the standpoint of Army aviation, there are two broad objectives to be met in solving rotor dynamics problems through a program of directed research. One objective is to reduce the magnitude and frequency of rotor dynamics-related failures and required redesign efforts in propulsion hardware development programs. Another objective is to improve the reliability and maintainability of future engines in the field through reduction of vibration and dynamic bearing loads.

The purpose of this paper is to report the current state-

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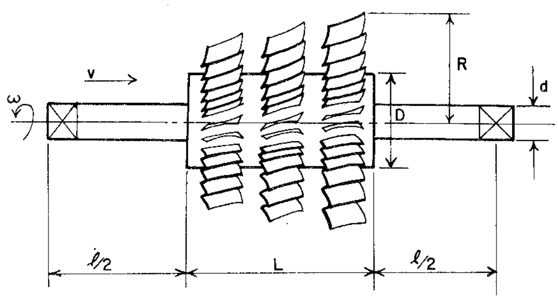

Fig. 1 Scaled dimensions.

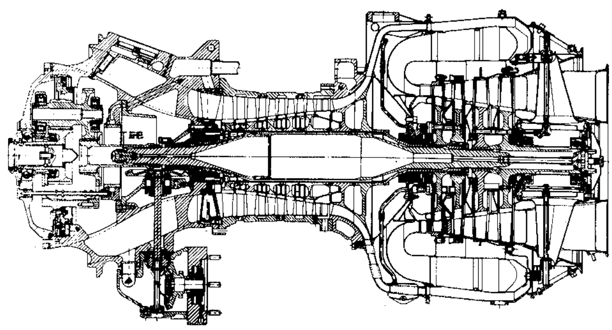

Fig. 2 Turbine engine cutaway.

of-the-art as completely and concisely as possible, drawing from extensive literature review, experience of the authors, and on-site visits to more than a dozen engine manufacturers, research houses, and universities, which are active in the technical field of rotor dynamics. An attempt was made to restrict the field of coverage to those aspects of rotor dynamics which are directly relevant or applicable to small turboshaft engines for helicopter or aircraft propulsion, although much of the information presented is applicable to rotor dynamics problems encountered in many different types of rotating machinery.

II. Scaling Factors

An appreciation for the effect on dynamics of physical size reduction can be gained through dimensional analysis of a rotor-bearing system. Consider a turbine or compressor wheel assembly (disks, spacers, and blades) centrally located on a flexible shaft, as shown in Fig. 1, and assume that it is desired to scale the system down in physical size without reducing airflow rates.

It has been the practice of turboshaft design engineers, with one or two recent exceptions, to maintain operating shaft speeds at least 20% below the first critical speed in shaft bending (usually the third critical speed). If the bearings of this example are rigidly supported, the bending critical speed is approximated by

$$\omega_{cR} = d^2\left[2.5\ E\Big/\left(M + \frac{\pi}{8}\rho d^2 \ell\right)\ell^3\right]^{1/2}$$
$$= 1.8(d/\ell^2)[E/(0.5 + \alpha_1{}^2\alpha_2)\rho]^{1/2} \qquad (1)$$

where E = Young's modulus, M = mass of wheel assembly, ρ = shaft mass density, $\alpha_1 = d/D$, $\alpha_2 = l/L$ and d, D, l, L are defined by Fig. 1. For a given material and geometric configuration, Eq. (1) can be rewritten as

$$\omega_{cR} = \bar{C}\,(d/\ell^2) \qquad (2)$$

where $\bar{C}$ is a constant.

In addition to these variables, others pertinent to a scaling analysis of the rotor bearing system are: ω = shaft speed, R = maximum blade radius, v = average axial air velocity, and Q = air volume flow rate. The most important dimensionless groups are

$$\pi_1 = \alpha_1(d/D,\ \ell/L)\ \text{etc.}$$
$$\pi_2 = \omega/\omega_{cR} \sim \omega\ell^2/\bar{C}d$$
$$\pi_3 = \omega R^3/Q \sim \omega R/v$$

To preserve similitude for performance prediction, all of these π groups must be held constant when engine size is reduced. The first π group determines the scale factor n.

$$(d_1/D_1) = (d_2/D_2)\ \text{requires}\ (D_1/D_2) = (d_1/d_2) = n \qquad (3)$$

where the subscript 1 refers to the large engine and subscript 2 refers to the small engine.

The second π group preserves the same critical speed margin (say 20%) in the small engine as in the large engine.

$$(\omega_1\ell_1{}^2/\bar{C}d_1) = (\omega_2\ell_2{}^2/\bar{C}d_2)\ \text{requires}\ \omega_2 = n\omega_1 \qquad (4)$$

The third π group required the same velocity flow profiles in the small engine as in the large engine. If, in addition, the same air volume flow rate is required (assuming equivalent gas temperatures) the third π group requires:

$$(\omega_1 R_1{}^3/Q) = (\omega_2 R_2{}^3/Q),\ \text{or}\ \omega_2 = n^3\omega_1 \qquad (5)$$

Clearly, Eqs. (4) and (5) are incompatible.

Engine aerodynamic performance requirements are usually given priority over dynamics requirements. Thus, the engine speed of the small engine, as dictated by Eq. (5), will be much higher than would be allowed by the critical speed margin, as dictated by Eq. (4). A very small reduction in engine size can completely eliminate a substantial critical speed margin.

For example, assume a 10% reduction in size of an engine with a 20% critical speed margin: Eq. (5) gives (n = 1.11) $\omega_2 = 1.37\ \omega_1$, also, $\omega_{cR(1)} = 1.25\ \omega_1$, so that $\omega_1 = 0.8 \times \omega_{cR(1)}$, then $\omega_{cR(2)} = 1.11\ \omega_{cR(1)} = 1.388\ \omega_1$, and $(\omega_2/\omega_{cR(2)}) = (1.37\ \omega_1/1.39\ \omega_1) = 0.99$. The margin has been reduced to only 1%, simply by scaling the engine down 10% in size.

The analysis and example is greatly simplified, and there are numerous other factors which must be considered in a realistic scaling analysis. Nevertheless, it does illustrate one of the basic problems of rotor dynamics in small engines. In fact, it is becoming ever more difficult to avoid supercritical operation (shaft speeds through and above bending criticals) in modern small high-speed turboshaft engines.

III. Effect of Front Drive Requirements

Another set of design conditions which is difficult to meet in small high speed engines is the combination of special requirements Eqs. (3) and (4), i.e., the front drive requirement coupled with the maintainability requirement. Referring again to Fig. 1, the critical speed Eq. (1) is derivable from the more basic equation:

$$\omega_{cr} = (k_s/m)^{1/2} \qquad (6)$$

in which k_s is the shaft stiffness effective at the disk (wheel assembly) and m is the effective mass at the disk. It is seen that maintaining a high critical speed depends on maintaining high shaft stiffness. The stiffness of a uniform shaft mounted on rigid supports, relative to a centrally applied load is

$$k_s = (48EI/\ell^3)$$

where I is the cross-section area moment of inertia, and the other symbols were previously defined. It is clear that the stiffness, and consequently the critical speed, is a very strong inverse function of the distance ℓ between bearings.

In a front drive turboshaft engine, either the distance between the bearings supporting the power turbine shaft cannot be shorter than the length of the compressor spool, or an intershaft bearing must be employed, since the power turbine shaft must pass through the inside of the compressor spool to reach the front of the engine (see Fig. 2). Intershaft bearings, supporting relative rotation between the compressor spool and the power turbine shaft, have a history of poor performance and maintainability. As a result of these considerations and constraints, some engines will have their power turbine shafts operating through and above a critical speed in shaft bending.

Safe operation of aircraft rotor-bearing systems at supercritical speeds can be obtained only when at least one of two conditions are reliably met: 1) Accurate high-speed balancing of rotor shaft assemblies. Passage through bending criticals may require multi-plane balancing (more than two planes). 2) Provision for significant amounts of external damping.

Satisfaction of condition 1 allows passage through critical speeds without destructive whirl amplitudes. Satisfaction of condition 2 also reduces whirl amplitudes, but additionally, it suppresses several mechanisms of dynamic instability, such as nonsynchronous whirl induced by internal friction, shaft stiffness asymmetry, or aerodynamic excitation. These phenomena will be discussed later.

The alternatives to a supercritical power turbine shaft in small high-speed front drive engines are: a) Intershaft bearings. b) Gas generator redesign to allow a large diameter power turbine shaft. c) Use of a power turbine shaft material with a significantly higher stiffness/density ratio than steel (e.g., beryllium).

The most commonly used method for keeping power turbine shafts subcritical is alternative a), the intershaft bearing. In all applications to date, antifriction bearings have been used for this purpose, although a design problem of radial space availability between the inner and outer shafts is often encountered. The most probable causes of the previously mentioned problems with rolling element intershaft bearings are shaft bowing from thermal effects, inaccessability for lubrication, and high DN values with counter-rotating shafts.

It is desirable from an aerodynamic standpoint to have the gas generator shaft and the power turbine shaft rotating in opposite directions, but this increases the DN value of the intershaft bearing. It is therefore a more common design practice to have these shafts corotating (although usually at different speeds). Intershaft bearings also appear to be a source of nonsynchronous vibration, since they can transmit dynamic loads from one shaft to the other at any of the predominant frequencies.

Another type of intershaft bearing which has potential merit is the oil film bearing. At least one engine manufacturer has attempted such an application without success. It should be noted that the load capacity of a film bearing is increased when the bearing corotates with the journal, and that film bearings have minimal radial space requirements. Of course, oil film bearings also have a much greater load capacity than gas bearings.

The second alternative b) to supercritical shaft design involves either an increase of gas generator bearing diameter (with a consequent increase in bearing DN values) to accommodate a larger diameter power turbine shaft, or a bias toward a purely centrifugal compressor design to allow a shorter shaft. Either of these options has the effect of raising the power turbine shaft critical speeds, as desired, but they both also tend to increase the cross-sectional size of the engine.

It may be of interest to note that a large diameter compressor spool bearing is compatible with some of the unique requirements of gas film bearings. For example, gas bearings require large bearing areas and high journal velocities to generate significant load capacity.

The final alternative c) has been the subject of some preliminary studies by at least one engine manufacturer. Returning to Sec. II, comparison of Eqs. (1) and (2) shows

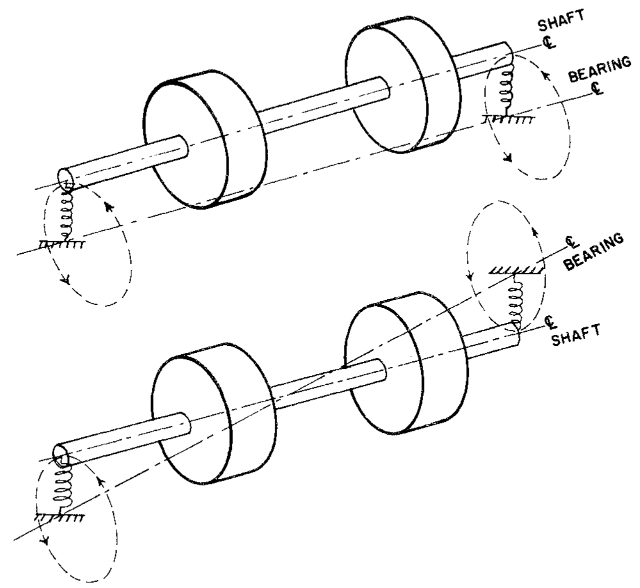

Fig. 3 Cylindrical whirl of a rigid rotor on flexible supports.

that the shaft critical speed is proportional to $(E/\rho)^{1/2}$. Most engineering metals in common use (e.g., steel, aluminum) have almost identical (E/ρ) values, thus offering little selectivity with respect to critical speed properties.

Beryllium is an exception to this rule, with an (E/ρ) value of about three times that of steel or aluminum. In designing a beryllium power turbine shaft, one problem to be overcome is the notch sensitivity and brittleness of the material. However, if these problems could be overcome through proper design or through modification of material properties, a power turbine shaft of this material could operate at significantly higher speeds (perhaps 20%) without passing through resonance in bending.

IV. Prediction of Critical Speeds

Critical speed analysis was historically and is still today the most important single rational method for rotor-bearing system design. It allows the designer to avoid resonant conditions in the operating speed range of his machine.

Referring to Eq. (6), the critical speeds of a rotor-bearing system are determined by the effective stiffness, which may be either in the supports or in the shaft itself, or both, and by the effective mass, which may be rotating with the shaft or vibrating with the bearing support structure. Equation (6) applies strictly only to a simple Jeffcott rotor[1,2] with a single critical speed, but the concept of effective stiffness and mass may be carried out to much more complex cases involving bending modes with distributed mass, in which these quantities must be regarded as speed dependent.

In rotor-bearing systems, the flexibility (inverse of stiffness) may be almost entirely in the bearing supports, in which case we say we have a "rigid rotor," or it may be almost entirely in the shaft, in which case we say we have a "flexible rotor." The corresponding critical speeds are called "rigid-body" critical speeds and "flexural" or "bending" critical speeds. The reason for the terminology is illustrated in Figs. 3 and 4.

Figure 3 shows the two modes of rigid body motion allowed by soft supports. The first mode, in which the two ends of rotor whirl in phase, is called "cylindrical whirl." The second mode, in which the two ends of the rotor whirl 180° out of phase, is called "conical whirl." Most modern turboshaft engines have bearing supports which are designed to be very flexible relative to the shaft and there-

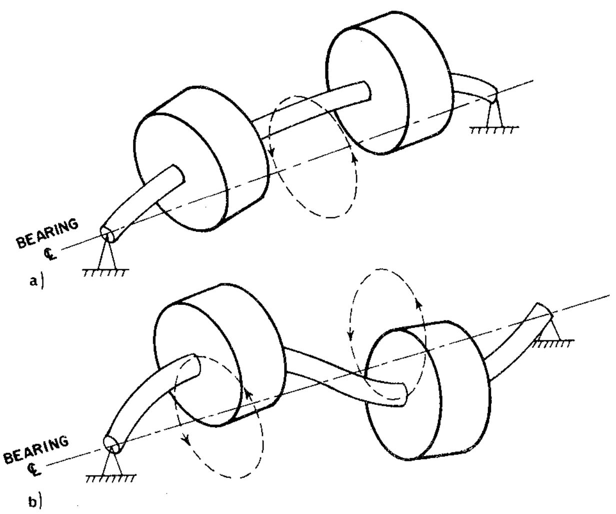

Fig. 4 Flexural whirl on rigid supports: a) first mode shape, and b) second mode shape.

fore pass through both of these rigid body criticals at speeds below the operating range of the engine. Since most of the motion in these modes takes place at the bearing support locations, the resonance can be damped by dissipating energy in specially designed dampers at the bearing supports.

If the supports were made rigid, the first mode would look like Fig. 4a, and the second mode would look like Fig. 4b. In this case there is no deflection of the bearing support structure, and since the shaft whirls in a constant bowed shape, it is difficult to damp these modes by dissipating energy in dampers. These flexural modes, with rigid supports, are therefore extremely difficult to pass through safely, unless the shaft is precisely balanced for the expected mode shapes.

At sufficiently high speeds, the rotor-bearing system of Fig. 3 also displays the flexural response of Fig. 4. Thus a rigid rotor changes into a flexible rotor just by an increase in speed. This is because all real shafts have some flexibility and will therefore have flexural resonances at sufficiently high frequencies, even with soft supports.

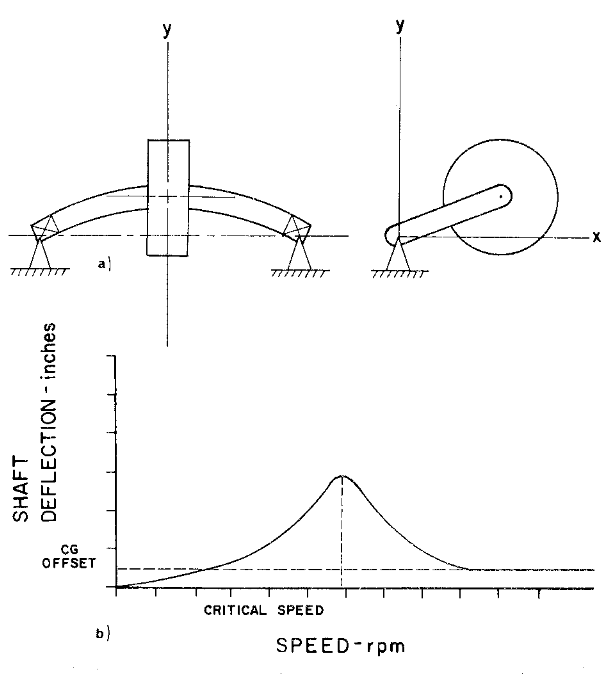

Fig. 5 Response curve for the Jeffcott rotor: a) Jeffcott rotor, and b) response curve.

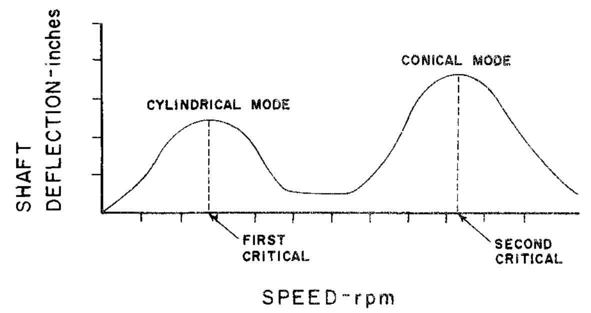

Fig. 6 Response curve for rigid rotor on flexible supports.

In the design of turboshaft (also turbojet, turbofan) engines which are to be used as a primary source of flight propulsion for aircraft, engineering design practice has almost always been to keep the flexural critical speeds above the maximum operating speed of the engine. The safety reasons for this should be obvious from the discussion above. The word "almost" is used because the definitions of rigid body and flexural critical speeds become hazy when the support flexibility and shaft flexibility are of about the same magnitude. Also, it can become very difficult to design a stiff power turbine shaft when the bearing span cannot be made shorter than the length of the compressor spool.

The original work by Jeffcott[1] was apparently the first published analysis of critical speed response which correctly predicted that shaft whirl amplitudes would come back down at speeds above the critical speed. A typical response curve for the simple rotor analyzed by Jeffcott is shown in Fig. 5. For this system, the effective stiffness and mass can be predicted quite easily, which allows accurate predictions of the critical speeds.

Note that the ordinate of the curve is shaft whirl amplitude, and the abscissa is shaft speed. The most important information displayed by such a curve is the speed at which the peak response occurs (i.e., the critical speed). The predicted whirl amplitude is less important because it is usually inaccurate. (This is because it is determined by the magnitude of system damping, which is extremely difficult to predict).

The response curve of Fig. 5 is also valid for the cylindrical rigid body mode of Fig. 3. However, if the response curve for the rigid rotor is extended out to a speed range which includes the conical critical speed also, the curve will display both peaks as shown in Fig. 6. (Note that the shaft deflection measured in the conical mode would depend on the measurement location along the shaft).

As the speed range of the rotor is further increased, more peaks will be displayed on the response curve, corresponding to the shaft flexural modes. The principal objective of all critical speed analysis is to determine the speeds at which these peaks occur, so that they can be adjusted outside of the operating speed range to proper design.

Although the complexity of modern turboshaft rotor systems has provided an incentive for development of critical speed analyses and computer programs with a high degree of mathematical sophistication, it is important to remember that the accuracy of the resultant predictions is completely dependent on the accuracy of the stiffness, mass, and damping data used as input to the calculations. In turboshaft engine design, one of the easiest rotor-bearing parameters for the engineer to adjust is the stiffness of the bearing supports. Therefore, the critical speed analysis is often used to generate curves like Fig. 7 (for example, see Ref. 3). Since a large number of critical speed calculations must be made to generate such a curve, it follows that the speed of computation can be an important factor in choosing a method of analysis.

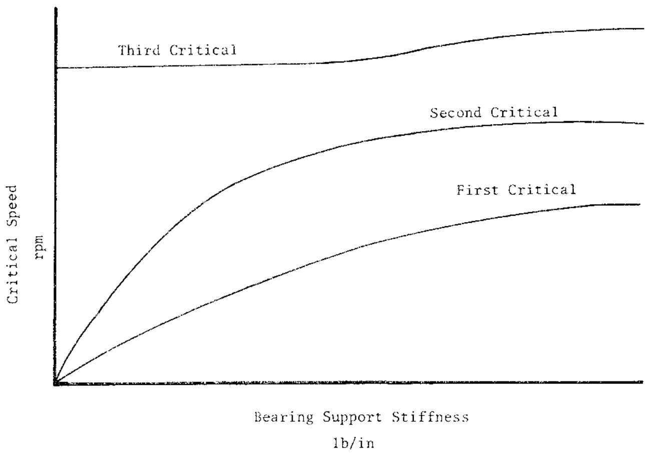

Fig. 7 Typical critical speed map.

There are five basic analytical methods (with many variations of each of the five) which have been developed to calculate critical speeds of rotor shafts. In their modern form, each has been adapted for use with high speed digital computers. All of these methods are applicable to rotor-bearing systems much more complex than the Jeffcott rotor, but each has certain advantages and disadvantages which make the choice of method dependent on the type of rotor, the speed range of interest, and the accuracy required. The methods are briefly described and referenced as follows:

1) The method of Stodola, now sometimes called the "matrix iteration method."[4,5] The calculation begins with an assumption of the first mode shape, from which the inertia loading due to whirling is calculated at an assumed critical speed. This loading is used to calculate the shaft deflection curve, which is compared with the assumed mode shape. If the agreement is not good, the process is repeated using the new calculated deflection curve and a properly adjusted value for the critical speed. A surprisingly small number of iterations will converge to the true first mode shape (eigenvector) and critical speed (eigenvalue). The calculation for higher order mode shapes is somewhat more complicated, but can be accomplished up through several critical speeds.

This method is especially well adapted to the use of influence coefficients, which can often be experimentally verified, thus adding to confidence in the results. The disadvantages of this method are a large requirement for computer storage capacity, and a loss of accuracy for the higher order modes. In graphical form, this was historically the first method used for turboshaft design analysis.

2) The Prohl-Myklestad method.[6,7,8] This method is similar to the Holzer method for torsional vibration analysis, in which the shaft is divided into a number of sections with the mass of each section concentrated at the ends. Beginning with the boundary conditions at one end of the shaft, the moment, shear, and inertial loading for each section are matched up with adjacent sections until the other end is reached. If the required boundary conditions at this end are not met then the calculation is repeated for another value of shaft speed (which determines the inertial loading). The critical speeds, which are the only speeds at which shaft deflection can exist without external loads or unbalance, are deduced from a plot of the end boundary conditions vs speed. The speeds at which the boundary conditions are met are the critical speeds.

This is presently the most commonly used method for critical speed analysis in the turboshaft engine industry. It has been recently modified[3,9] to include damping effects and to improve computational efficiency.

3) The Rayleigh-Ritz method,[10] also known as "the energy method." The maximum strain energy in the rotor shaft is equated to the maximum kinetic energy due to whirling. Since the kinetic energy is a function of shaft speed, the resulting equation can be solved for the critical speed.

The main disadvantage of this method is that the calculations for strain and kinetic energy require the shaft deflection shape in the desired mode, which is generally not known and must therefore be assumed. The accuracy of the method is not, however, very sensitive to errors in this assumption, and there are parametric variation methods available to minimize the error.

4) The characteristic equation method.[11] Substitution of a general exponential solution into the differential equations of motion yields a polynomial in the eigenvalues (critical speeds), the roots of which are the critical speeds. This method is not presently used much in the industry, probably because the polynomials obtained for real systems are of high order and therefore difficult to solve. There has been some recent interest in applying modern algebraic techniques to update this method, however.[12]

5) The method of numerical integration, also called the marching method.[13,14] The equations of dynamics for the rotor-bearing system are solved numerically, marching out the motion from the initial conditions for small steps of increasing time.

This method consumes large amounts of computer time, since the critical speeds are obtained by calculating steady-state amplitudes for a large number of speeds and plotting the results as in Figs. 5 or 6. Sufficient computer time must be allowed at each speed for initial transients to die out.

It is, however, the only method which can simulate the nonlinear system. It is therefore valuable for verification of the results from methods 1–4 and for investigation of the effect of nonlinearities on critical speeds.

As can be inferred from the aforementioned, critical speed analysis is now advanced to a highly sophisticated state, with a number of techniques developed to handle practically any type of rotor-bearing system. The references noted were selected to give some description of the basic concept and theory for each method.

Each of the turboshaft engine manufacturers and industry-related research houses have their own highly developed computer programs for critical speed analysis based on these methods. Only a few of these programs are well documented in the published literature,[15,16] however, which tends to perpetuate a certain lack of comparability of results, even for similar problems.

Gyroscopic effects can be included in all of the critical speed analyses listed above, and practically all of the industry computer programs do include these effects. This is usually done by treating compressor and turbine wheels as if they were rigid disks. Until recently, this was a valid assumption since these disks are normally very rigid compared to the shaft.

Some recent problems in making accurate critical speed predictions for extremely high speed ($N > 60{,}000$ rpm) turbomachinery, and for compressor rotors with long thin blades, tend to suggest that there is a class of machines for which the rigid disk assumption is not valid.

The extremely high out-of-plane forces developed on whirling disks at high speed apparently can bend the disk and thus modify the gyroscopic moments. The suggested type of deformation is shown in Fig. 8. The terminology "rubber disk effect" was apparently coined by J. P. Den Hartog during consultations with industry companies on this problem. With the exception of the "rubber disk effect," critical speed analysis appears to be sufficiently well developed to treat all foreseeable cases in turboshaft rotor-bearing design. Any significant advances to be made in these analytical capabilities would most likely be to

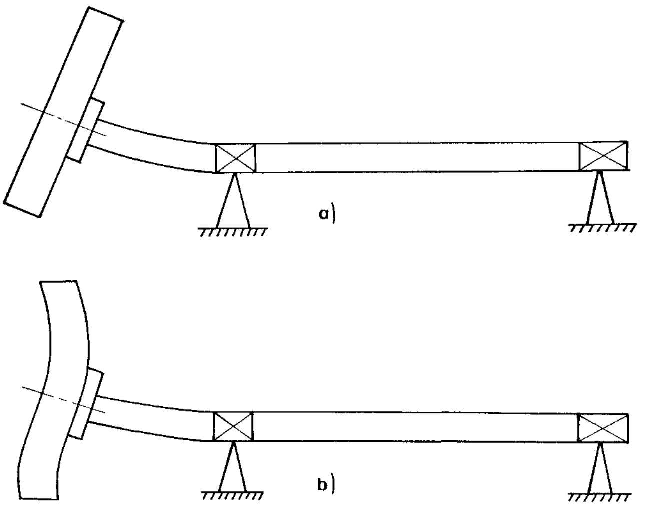

Fig. 8 Disk deformation affecting gyroscopic moments: a) rigid disk on flexible shaft, and b) flexible disk on flexible shaft.

improve computational efficiency and reduce computer time costs. The real need is for more accurate determinations of rotor system parameters and characteristics to be used as input to the various computer programs.

Turboshaft engine rotors are actually an assembly of shafts, spacers, disks, and blades. In many cases, the methods of attachment of these pieces are governed by maintainability considerations which require a disassembly capability, or by thermal expansion clearance requirements which are often satisfied by the use of spline joints.

Critical speed calculations require knowledge of the bending stiffness of the rotor shafts in the system at all points along the shaft. The effects on bending stiffness of the various joints and discontinuities are largely unknown at the present time. Even with welded joints, which are becoming more common in modern engines, the shaft stiffness often cannot be accurately predicted in the design stages. In practice, engineering design analysts make "educated guesses" based on experience for these stiffnesses, and later improve the estimates by component testing or by critical speed measurements.

V. High-Speed Balancing

For a completely rigid rotor, it has been shown that two balance planes (locations along the shaft at which unbalance measurements and corrections are made) are both necessary and sufficient for balancing to be effective at all speeds.[17] Rotor shafts which flex obviously have a state of balance which changes with the magnitude and shape of shaft deflection. There has been some controversy among the experts as to exactly how many balance planes are required for flexible rotors, but is fairly certain that more than two are required,[17] and that no more than $N + 2$ are needed,[18] where N is the number of critical speeds to be passed through.

One current program of research and development for high speed balancing of flexible rotors is based on an influence coefficient method, in which the required correction weights and locations are calculated from experimental data obtained with the rotor running at various speeds with a known trial unbalance weight attached. References 19 and 20 report test verification of the theory, which is based on the work of Goodman,[21] Rieger,[22] Lund and Tonnesen.[23] Application of the method to a high speed turboshaft engine is described in a publication by Rieger and Badgley.[24] A similar method for flexible rotor balancing which uses influence coefficients that are calculated from beam theory, rather than experimentally measured, is reported by LeGrow.[5]

Another method for flexible rotor balancing is the modal method,[18] in which balance planes are selected to have maximum influence on particular flexural modes, and the balancing is done at the flexural critical speeds to minimize the effect of other modes. Whatever method eventually proves to be best for small turboshaft engine applications, it appears that balancing in more than two planes will become necessary as engine design evolves further into the region where flexural whirl modes are significant.

Flexible rotor balancing requirements will also have their own effect on engine design. The significant factor is that balancing flexural modes must be done either in the engine, or on specially constructed bearing supports which simulate the engine bearing stiffnesses and locations.

The latter approach might be simplified for some applications using very soft supports (without squeeze film dampers) by mounting the rotor "free-free" in a specially designed balance machine. In any case, however, balancing the rotor outside the engine would be compromised by the necessity for disassembly and reassembly on installation, unless there is a major change in design philosophy to allow installation of assembled rotors in engine cases after balancing. "In-Place" balancing (balancing complete assembled rotors in the engine) would also require a major change in design philosophy to provide integral probes for measurement of rotor deflection and access ports for balance correction.

There is a potential side benefit from in-place balancing which could prove more valuable than the balancing itself. This is the diagnostic capability provided by the integral proximity probes. Manufacturers of high speed compressors and other types of rotating machinery have recently begun to use such probes as a routine source of information on bearing conditions, state of rotor balance, etc.

For turboshaft engine applications, proximity probes in the hot gas sections would be subjected to an extremely severe environment in terms of temperature and erosion. At present, probes are not available to survive this environment, but they are under active development and should be available within one to two years from the date of this paper.

VI. Prediction and Control of Bearing Support Properties

The two bearing support properties of interest are stiffness and damping. As described previously, most modern engines are designed with soft supports (low stiffness) to place the rigid rotor critical speeds below the operating range. Support damping can reduce or eliminate the peaking synchronous response to unbalance at the critical speeds (resonance), and can suppress many of the nonsynchronous responses and associated instabilities.

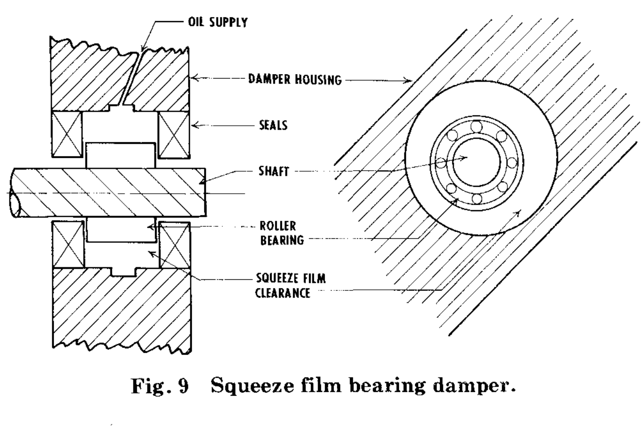

Fig. 9 Squeeze film bearing damper.

Asymmetry and cross-coupling are important aspects of support properties, as they have a profound effect on rotor dynamics, especially with regard to stability. An example of asymmetry is different bearing support stiffnesses in the horizontal and vertical directions. An example of cross-coupling is damping force generated in the horizontal direction by motion in the vertical direction. The influence of support properties on rotor shaft dynamics is now well understood by researchers in the field, the subject having been extensively studied both analytically and experimentally.[25-31]

The real problem at present is a lack of capability to reliably and accurately predict support characteristics from design data. The squeeze film bearing damper represents an attempt to achieve a high level of support damping, and is probably the most significant development of the last decade affecting high-speed rotor dynamics. Figure 9 illustrates the principle of operation.

The terminology "squeeze film" is descriptive of what takes place in the annular clearance space, to which oil is continuously supplied. In a typical turboshaft application, the clearance space is located between the damper housing bore and the outer race of a rolling element bearing, which is therefore a loose fit in the damper housing. The rotating unbalance of the rotor induces the bearing to orbit within the damper housing. In a properly designed damper, metal to metal contact is prevented by hydrodynamic support of the oil film. The bearing race is normally constrained by a key to prevent rotation.

The oil film support provides both low dynamic stiffness and high damping (energy dissipation). The result, at least for a good design, is greatly reduced dynamic bearing loads, elimination of resonance, and in some cases even a reduction of whirling amplitudes. The latter result is intuitively surprising to many; it is often difficult to convince a machine designer that he needs to provide a loose bearing housing clearance to reduce whirling amplitudes.

In this description, the phrases "properly designed damper" and "good design" are important. At present it is not possible to predict damper performance from design data with confidence. The good designs that have been obtained are largely based on empirical information from earlier designs. The same can probably be said about the bad designs.

A squeeze film damper is basically an oil film bearing with zero rotation. The damper force response should therefore be predictable from hydrodynamic bearing theory. For example, Lund[32] has defined eight coefficients which give the forces in terms of journal position and motion in cartesian coordinates.

The problem is that these coefficients are predicted differently by each of the various theoretical models based on certain assumptions for analytical simplification. There is "long bearing" theory,[33] and "short bearing" theory,[34] and various assumptions about the circumferential location of the beginning and end of the oil film (the boundary conditions). Most experimental data for oil film bearings has been obtained for the case with pure rotation, no orbiting, which is the antithesis of a squeeze film damper.

Whenever an attempt is made to design a damper on a scientific or analytical basis, it is usually based on the work of Cooper,[35] who gives mostly qualitative results of a parametric study made with an unbalanced rotor supported by a squeeze film. The most convincing and indisputable fact shown by Cooper's experiments is that any restriction of damper orbiting by mechanical stops or bumpers produces a tremendous increase in bearing loads, perpetuates resonance which would not occur without the bumper, and generally destroys the good effects of the damper.

More recently, Jones[36] published the results of an experimental study of squeeze film hydrodynamics. He reports a reasonable verification of short bearing theory except for cases with large eccentricity (bearing journal far off center). The main shortcoming of this study is that the apparatus motion was restricted to orbiting about the damper centerline, a condition rarely obtained in engines unless the damper is coupled with a stiff mechanical support spring.

Since the squeeze film damper is a nonlinear device, the possibility exists for several shaft motions to satisfy dynamic equilibrium. In fact, an analytical and experimental investigation by White[37] has confirmed the existence of jumps from one stable orbit to another of different magnitude, a phenomenon originally suggested by Cooper's work.[35]

Some manufacturers and turbomachinery research houses have developed computer simulations of the transient dynamics of engine rotors on squeeze film supports.[13,14] Although such simulations are helpful in understanding the effects of changes in various design parameters, little confidence can be placed in their quantitative predictions until the hydrodynamic response portion of the analytical model is either verified or appropriately modified by experimental studies.

Understandably, there are some engineers in the industry who prefer not to rely on the imprecisely known characteristics of squeeze film dampers to provide the required support properties for control of rotor dynamics. Generally speaking, the stiffness of mechanical supports is easier to control, even if not so easy to predict in the preliminary stages of design. The damping of such supports is, of course, very predictable, usually being very low. Even when squeeze film dampers are used, they often are mounted in series or parallel with mechanical spring supports.

Mechanical bearing supports are usually designed to provide the minimum stiffness practical while still maintaining the required strength and reliability. Most fall roughly into one of three categories: a) The squirrel cage. So named because of its appearance, made as a cylinder of thin metal ribs. b) Welded rod support. In this design the bearing mount is cantilevered on several metal rods parallel to the shaft centerline. c) Corrugated metal ring. A variety of these designs are all constructed to fit snugly around the outer bearing race and provide a mechanical cushion through deflection of small segments or protruding elements.

The stiffness properties of all these designs can be controlled with good accuracy by testing and iterative redesign as required. The capability to predict these properties analytically in preliminary design stages needs to be improved. It appears that one way of doing this would be to standardize some designs throughout the industry, although the practicality of accomplishing this in a competitive environment is questionable.

Another type of mechanical bearing support, which is used to provide both low stiffness and some degree of damping, is the common "O-ring." Grooves in the bearing housing bore are made to hold the "O-rings" so that they are compressed into an elliptical shape when the roller bearing outer race is inserted.

The "O-rings" are made of an elastomeric material, and their fatigue properties and thermal degradation properties are not well known for this type of application. At present they are considered suitable only for short life applications in the cold (compressor) section of turboshaft engines.

VII. Nonsynchronous Excitation

Rotor-bearing system response to rotor unbalance is called synchronous whirl, because it is characterized by rotor orbiting at shaft speed. Synchronous whirl is usually in the same direction as shaft rotation (forward), but can also be backward in direction.

All other motions executed by the rotor bearing system can be classified either as nonsynchronous whirl or as nonsychronous vibration. These motions are often self-excited by rather subtle mechanisms, and are sometimes associated with the possibility of dynamic instability.‡ It is this latter characteristic which makes an understanding of nonsynchronous excitation so important to insure safe designs for flight propulsion.

The major sources of nonsynchronous excitation which have been identified and studied by researchers to date are: 1) Asymmetric stiffness properties of shafts or bearing supports.[26-29,31,38-41] 2) Internal friction in shafts or other rotating parts.[26,42-46] 3) Aerodynamic or gas flow excitation.[48,49] 4) Excitation from fluids trapped within rotor shafts.[50-52] 5) Rubbing friction between radial surfaces or rotors and stator housings.[47,53] 6) Excitations from other rotating components in the same engine structure (e.g., excitation of a power turbine rotor from unbalance of a compressor rotor). 7) Excitations from gears, at the gear tooth mesh frequencies.

Backward whirl, which can be either synchronous or nonsynchronous, is one of the least well understood phenomena of rotor dynamics. Den Hartog[54] states in his book that he doubted the possibility of its existence until he finally observed it in a model. Although Den Hartog concluded that backward whirl is only of minor importance, more recent experience in the turboshaft engine industry indicates otherwise. Large amplitudes of backward whirl have been observed in engines, with the source of excitation traced to the ball bearing effects originally identified by Yamamoto.[55]

Backward whirl has been shown to be an eigenvalue (natural frequency) of rotor bearing systems in a number of critical speed studies.[3,11,31,56] Since any shaft unbalance always rotates forward with the shaft, backward whirl would not seem to be excited by unbalance. It has been shown, however, to be excited when the bearing support stiffness is asymmetric,[57] and is also believed to be associated with gyroscopic moments.[58] References 2 and 56 are in disagreement as to the possibility of the existence of backward whirl when damping is present in the system.

Radial rub between rotors and stator housing is the most easily visualized mechanism driving backward whirl, and this may actually be the source of most cases occurring in turboshaft engines. Inspection of Army helicopter engines being rebuilt at ARADMAC provides convincing proof that such rubs are frequent and pronounced in these engines.

Nonsynchronous whirl inducted by internal friction is probably the most common type of self-excited whirl in turboshaft engines. The nature and cause of this phenomenon has been rigorously analyzed by Gunter[44] and Ehrich[42].

A subsequent investigation by Gunter and Trumpler[45] shows that an increase in the threshold speed of instability can be provided by asymmetric bearing supports. These analyses also confirm earlier experimental findings that both increased bearing support flexibility and external damping raise the threshold speed of instability. Since squeeze film dampers provide both of these effects simultaneously, it is to be expected that friction-induced whirl could be suppressed by their use.

The first author has investigated the effect of rotor unbalance, shaft stiffness asymmetry, and the location of external damping in the system, on friction-induced whirl.[46] It was found that aerodynamic drag can be an important source of external damping to suppress friction-induced whirl, whenever stiff bearing mounts are used. This is because bearing supports cannot dissipate significant amounts of energy unless they are flexible enough to move.

The most important parameter affecting the threshold speed of instability for friction-induced whirl is the ratio of internal friction to external damping. For large values of this parameter, rotor-bearing operation is unstable at all speeds above the critical speed associated with the motion producing the internal friction. For values of about unity (external damping equal to internal friction), the threshold speed is about twice the critical speed. Thus it is seen that, with a reliable and effective mechanism for external damping, rotor-bearing systems could be safely operated at speeds up to about 80% above the critical speed.

The problem in turboshaft engine design is that neither external damping nor internal friction can be reliably predicted from design data, or indeed, even after hardware already exists. At present these variables can only be measured indirectly from their dynamic effects on the system.

There are numerous potential sources of internal friction in a typical turboshaft engine rotor assembly. In addition to hysteresis of the material itself, shaft splines, shrink fits, and bolted connections are all capable of generating friction forces as the rotor-shaft assembly flexes. Some experimental data giving at least the relative magnitudes of friction generated by these mechanisms could be very helpful to the rotor dynamics engineer for preliminary design analysis.

Rotor whirling induced by aerodynamic forces on blades and seals may also be nonsynchronous, and is such a complex phenomenon that research to date has been confined mostly to hypothesizing qualitative theories to explain it.

It can be surmised that an analogy to "oil whip" in hydrodynamic bearings might exist for bladed disks in cylindrical housings with small tip clearances, or that a similar analogy to "propeller whirl flutter" might exist for compressor stages with long blades. What is needed is combined experimental measurements of rotor motion and circumferential pressure distributions to establish the relationships that exist between disk whirling and aerodynamic forces.

Nonsynchronous whirl appears to occur more frequently in rotors with cantilevered disks. In fact, there has been a long history of dynamics-related problems associated with overhung rotor disks which are almost certainly not associated with synchronous response to unbalance.

Figure 8 illustrates the overhung rotor disk configuration which is characterized by a disk having shaft bearings on only one side. The problems referred to have been recognized by numerous investigators, and there is a considerable body of literature addressing the subject.[39,40,58,59]

VIII. Hydrodynamic Bearings

Hydrodynamic bearings provide rotor shaft support through pressure in a thin fluid film between the shaft journal and bearing. The fluid may be either a gas (usually air) on a liquid (usually oil). The pressure may be self-generated from rotation of the journal (wedge effect) or may be supplied externally from an auxiliary pump or compressor. The chief distinguishing characteristic of hydrodynamic bearings, as opposed to boundary-lubricated bearings, is that metal to metal contact between the journal and bearing does not (or at least should not) occur except possibly during startup or shutdown.

Rolling-element bearings are firmly entrenched in aircraft turboshaft engine design, and there is only one case known to the authors in which a hydrodynamic bearing has been successfully used for this application. The two

‡Dynamic instability is defined here as a motion which becomes unbounded (until system limits are exceeded) either with time or with some normally variable parameter of the system, following an initial perturbation.

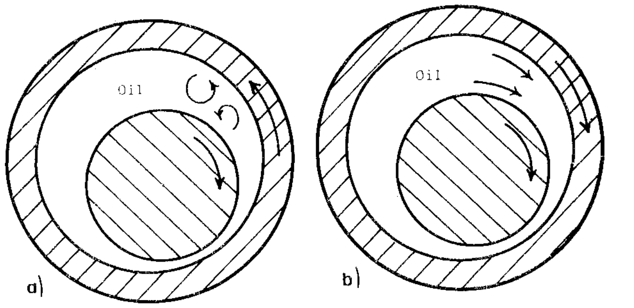

Fig. 10 Dependence of intershaft bearing load capacity on rotation direction of both shafts: a) journal and bearing counter-rotating, turbulent eddies induced in oil film, no support pressure generated; b) journal and bearing co-rotating, oil drawn into converging wedge to generate pressure.

principal reasons for this are: 1) a rolling-element bearing usually fails in a gradual way, which gives warning time before aircraft power is lost, and 2) rolling-element bearings reject less heat to the lubricant, thus allowing a smaller heat exchanger for cooling.

The latter comparison does not apply to gas bearings. This, and the reliable availability of lubricant for a gas bearing, has provided considerable incentive for research and development of gas bearings for turboshaft applications.

The chief advantage which all hydrodynamic bearings offer is long life. In addition, gas bearings also offer extremely low friction coefficients, although they have a much smaller load capacity for their size than fluid film bearings.

Reference 60 describes a feasibility study to apply gas bearings to a small turboshaft engine. Dynamic stability problems were encountered in the attempted application, which is a common occurrence with gas bearings. More recently the Army and the Air Force Aero-Propulsion Laboratory have supported work to develop gas bearings for application to gas turbines.

Oil film bearings may also have a place in small turboshaft engines of the future. Although rolling element bearings are called "anti-friction" bearings, they do not necessarily have a lower friction coefficient than oil film bearings. Furthermore, in some stationary applications, oil film bearings have demonstrated extremely long life capabilities. In the one successful turboshaft application of an oil film bearing, mentioned earlier, the incentive was a limitation of radial space.

The required heat exchanger size, and flight safety considerations, probably preclude broad application of these bearings in turboshaft aircraft engines. Nevertheless, it is likely that there will be special applications where long life or radial space is a problem which can best be solved by an oil film bearing.

Problems of dynamic instability with oil film bearings, such as "oil whip,"[33] have been effectively solved through research and development of new bearing designs. An example is the "tilting pad bearing," in which the cylindrical bearing sleeve is replaced with several pivoted blocks around the circumference, each one supporting the journal over a segment of the cylindrical surface. In addition to being more stable, this type of bearing can also accept greater shaft misalignment.

Due to limitations of radial space between the compressor spool and power turbine shaft, described earlier, oil film bearings may find useful application in this location as intershaft bearings. In fact, this has been attempted in one engine development program which the authors are aware of. First tests were not successful, and the idea was abandoned without determining the cause of failure.

Intershaft oil film bearings require corotating shafts (same direction), since the hydrodynamic support is greatly reduced in counter-rotating shafts, and completely disappears if the shafts rotate at equal speeds in opposite directions. Figure 10 illustrates how the wedge effect to generate support pressure depends on rotation direction when both the journal and the bearing are rotating.

Whenever a hydrodynamic bearing is used in aircraft propulsion machinery, the possibility of lubricant supply interruption or failure must be considered in the design. Both military specifications and FAA requirements demand some operating time after lubricant supply interruption. One way to approach this problem is to design lubricant reservoirs near the bearings.

Hybrid bearings, in which a hydrodynamic bearing is located inside a rolling element bearing, are under development at the NASA-Lewis Research Center.[61] The hybrid bearing may prove to combine some of the best features of both types. Since the film bearing and rolling element bearing are in series, the result is a corotating journal and bearing for the former and a smaller DN value for the latter.

IX. Summary

Higher shaft speeds, combined with the desire for front drive, good maintainability, and long life, have created some challenging problems for the rotor dynamics engineer in the turboshaft engine industry. The problems are intensified by scale effects in the smaller engines.

Critical speed analysis has been developed to a high state of refinement, and is not a restricting factor in engine design. Methods for high-speed balancing through several flexural critical speeds are under development, and it may become necessary to modify engine design philosophy to accommodate these methods if speeds continue to increase.

Squeeze film bearing dampers have contributed to smoother operation of engine rotors, make passage through critical speeds safer, and help to suppress potential whirl instabilities. At present, however, damper design cannot be optimized because an experimentally verified theory of dynamic force response does not exist.

Experimental data is needed on the stiffness and friction properties of the types of joints commonly used in rotor shaft assemblies, including splines. Rolling-element bearings have been and will continue to be the predominant type of bearing used in aircraft turboshaft engines. However, oil film and gas film bearings appear to be well suited for intershaft applications in front drives for these engines, and current development efforts for this purpose show promise.

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Dynamic Response of Viscous-Damped Multi-Shaft Jet Engines

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Rotor synchronous vibration experienced on multi-shaft aircraft engines results directly from rotor imbalance exciting the numerous critical speeds inherent in light-weight, high-speed modern powerplants. The understanding and reduction of this dynamic response is essential during engine design and development phases. This paper presents an efficient analytical technique capable of predicting the vibratory response of an engine with nonlinear viscous damping. A unique transfer-matrix method is applied to the idealized equivalent engine system to produce an unusually small array of influence coefficients. The damper equations for a closed-end viscous damper are derived from the basic Reynolds equation. The analysis is applied to a two-shaft aircraft engine to illustrate the basic concepts of multi-shaft critical speeds and nonlinear viscous-damped response.

Nomenclature

c = damper radial clearance
e = damper eccentricity
g = gravitational constant
h = damper film thickness
k_t = shear stiffness shape factor
l = length of beam element
m = mass
t = time
u = imbalance
x, y, z = rectangular coordinates
A = area
A_{ij} = influence coefficients
B = damping coefficient
D = damper journal diameter
E = elastic modulus
G = shear modulus
I = area moment of inertia
I_g = $(I_P - I_T)g$
I_P = polar moment of inertia
I_T = transverse moment of inertia
K = stiffness
L = length of damper journal
M, N = beam moment components
M' = moment resulting from imbalance force
P = damper oil film pressure
Q = state variables (shear moment, slope, deflection)
Q' = state variables resulting from imbalance force
R = damper journal radius
S = Sommerfeld number
T = torsional stiffness
T = moment imbalance
U, V = beam shear components
U', V' = beam shear components resulting from imbalance forces
U_y = imbalance in y-plane
U_z = imbalance in z-plane
Y, Z = beam deflection components
α = influence coefficients
β = boundary condition coefficients
ϵ = e/c (damper attitude)
η = phase angle of applied moment imbalance
θ = angle from line of centers, viscous damper
θ, γ = beam slope components
μ = oil viscosity
ν = mass/unit length
ϕ = phase angle of applied force imbalance
ϕ = viscous damper phase angle
ω_1 = inner journal spin speed
ω_2 = outer journal spin speed
ω = spin speed
Ω = whirl speed
$*$ = starred state variables represent values on left of massless beam

Subscripts

n = mass stations
L = line
N = span

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Introduction

THE importance of rotor dynamic technology has increased significantly during the past five years, and today this technology impacts the gas-turbine engine bearing configuration, the development program, certification requirements, and manufacturing and assembly procedures. A definition of the engine critical speeds and steady-state response to inherent rotor imbalance is required to allow the engineer to optimize the rotor and case structure for minimum weight and sensitivity to imbalance. The complex vibration response which results from a multishaft, high-speed, lightweight engine structure must be conveyed to the design engineer in a manner that allows easy manipulation of the design variables in order to produce the best structure with current technology.

The technical literature contains numerous rotating-machinery forced-response analyses which are extensions of the general theory of transfer matrix methods for transverse vibration as originally presented by Myklestad[1] and Prohl.[2] Prohl applied classical beam theory to rotating shafts and calculated the simply supported natural frequencies (critical speeds) considering symmetric, flexi-